ITERATIVE SCHEMES FOR HIGH ORDER COMPACT DISCRETIZATIONS TO THE EXTERIOR HELMHOLTZ EQUATION

Yogi Erlangga and Eli Turkel

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**Boundary conditions**

Suppose that at the boundary corner we have a simplified radiation condition of the form

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

where the directional derivative along the vector is defined as:

|  |  |  |
| --- | --- | --- |
|  |  | (2) |

We approximate the directional derivative by

|  |  |  |
| --- | --- | --- |
|  |  | (3) |

where the -th and -th grid points lie outside the computational domain, and hence are virtual points.

Differentiating (1) with twice we obtain a relation for the third-order and fifth-order derivatives in (4):

etc… =>

|  |  |  |
| --- | --- | --- |
|  |  | (4) |

Substitution of the above relation into (3) yields

|  |  |  |
| --- | --- | --- |
|  |  | (5) |

|  |  |  |
| --- | --- | --- |
|  |  | (6) |

|  |  |  |
| --- | --- | --- |
|  |  | (7) |

and hence from (1),

|  |  |  |
| --- | --- | --- |
|  |  | (8) |

|  |  |  |
| --- | --- | --- |
|  |  | (9) |

|  |  |  |
| --- | --- | --- |
|  |  | (10) |

|  |  |  |
| --- | --- | --- |
|  |  | (11) |

We now assume that the left-hand-side average is partitioned such that (this assumption should be verified):

|  |  |  |
| --- | --- | --- |
|  |  | (12) |

=>

|  |  |  |
| --- | --- | --- |
|  |  | (13) |

or

|  |  |  |  |
| --- | --- | --- | --- |
| |  |  |  | | --- | --- | --- | |  |  | (14) | |